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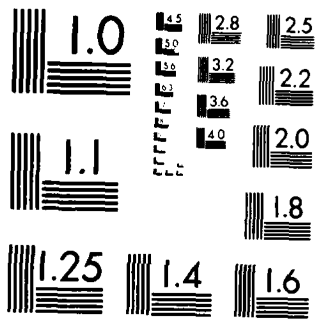
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DEGREE OF ADJACENCY OR SURROUNDEDNESS

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Definitions of the degree of adjacency of two regions in the plane, and the degree of surroundedness of one region by another, are proposed. Some elementary properties of these concepts are established, and it is also shown that they have natural generalizations to fuzzy subsets of the plane. Applications of the proposed measures to digital polygons are demonstrated and fast algorithms for computing these measures are given.

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ABSTRACT

Definitions of the degree of adjacency of two regions in the plane, and the degree of surroundedness of one region by another, are proposed. Some elementary properties of these concepts are established, and it is also shown that they have natural generalizations to fuzzy subsets of the plane. Applications of the proposed measures to digital polygons are demonstrated and fast algorithms for computing these measures are given.

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n	0	1	2	3	4	5	6	7	8	9	10
$a^*_{dig}(S,T_n)$	.00000	.30703	.41746	.30439	.29371	.24193	.30531	.32677	.19858	.14743	.11859
$a^+_{dig}(S,T_n)$	.00000	.18203	.16746	.17895	.16871	.24193	.18031	.32677	.19858	.14743	.11859
$a^*_{dig}(T_n,S)$	.00000	.08333	.22222	.17936	.17936	.15749	.19363	.21051	.13227	.09940	.07870
$a^+_{dig}(T_n,S)$	.00000	.00000	.05555	.09603	.09603	.15749	.110299	.21051	.13227	.09940	.07870

Table 1. Results for Example 2.1.

1. Introduction

In describing a picture, one often needs to specify geometric relations among the regions of which the picture is composed. For a review of such relations and their measurement in digital pictures, see [1].

Adjacency is an important relation between regions. In a digital picture, sets S and T are adjacent if some border pixel of S is a neighbor of some border pixel of T; in the Euclidean plane, regions S and T are adjacent if their borders intersect. Note that this relation is not quantitative; S and T are not considered adjacent even if they are very close to one another, and it also makes no difference whether they are adjacent at one point or at many points. In Section 2 we propose a quantitative definition of adjacency which does take these factors into account.

Another important region relation is surroundedness. We assume that all pictures are of finite size; the region of the plane outside a picture is called the background. We say that S surrounds T if any path from T to the background must intersect S. This definition too is nonquantitative. In Section 3 we propose two ways of defining the degree to which S surrounds T.

2. Quantitative adjacency

In Section 2.1 we define quantitative adjacency for regions in the Euclidean plane. In Section 2.2 we generalize this definition to fuzzy subsets of the plane, and in Section 2.3 we discuss quantitative adjacency of subsets of a digital picture.

2.1. Euclidean regions

Let C_0 be a rectifiable simple closed curve in the plane, and let $C_1,\ldots,C_n$ be rectifiable simple closed curves not crossing one another and contained in the interior $\hat{C}_0$ of C_0. According to the orientation of C_0, the closed set $C_0 \cup \hat{C}_0$ will be either a bounded set, or the infinite plane except for a bounded set (infinite case). For $C_1,\ldots,C_n$ we assume that $\hat{C}_1, \ldots, \hat{C}_n$ are bounded sets. Then $(C_0 \cup \hat{C}_0) - (\hat{C}_1 \cup \ldots \cup \hat{C}_n)$ is called a <u>region</u>; C_0 is called its <u>outer border</u>, and $C_1,\ldots,C_n$ are called its <u>hole borders</u>. Note that in the infinite case, there is no unique distinction between the outer border and the hole borders because the outer border may be considered to be a hole border itself. The <u>perimeter</u> of the region is the sum $\sum_{i=0}^{n} |C_i|$ of the lengths of its borders.

Intuitively, two regions S and T are (somewhat) adjacent if some border of S (nearly) touches some border of T; the degree of adjacency depends on how nearly they touch and along how much of their lengths they do so. The borders nearly touch if they are close to one another, as illustrated in Figure 1a-b; note that S and T are allowed to overlap.

Note, however, that not all cases in which borders are close to each other imply near-adjacency, as shown in Figure 1c. The difference is that in Figures 1a-b, the shortest paths between the close borders lie outside both regions or inside both of them, while in Figure 1c these paths lie inside one region and outside the other. It also seems plausible that only line-of sight paths should be counted in defining adjacency; in Figure 2a, the left-hand edge of T should contribute to its degree of adjacency to S, but its other edges should not, and similarly in Figure 2b, the parts of the border of the concavity in T from which S is not visible should not contribute. Finally, note that quantitative adjacency is not symmetric; in Figure 3(a-b), S is highly adjacent to T, since much (or all) of its border nearly coincides with the border of T, but T is not as highly adjacent to S, since only a small fraction of its border coincides with that of S.

Based on these considerations, we define the degree of adjacency of S to T as follows: Let P,Q be any border points of S and T, respectively. If $P \neq Q$, we say that the line segment $\overline{PQ}$ is admissible ("with respect to (S,T)" understood) if its interior lies entirely outside both S and T or entirely inside both of them. If P=Q, we call $\overline{PQ}$ admissible if the (signed) normals to the borders of S and T at P do not point in the same direction. Let d_P be the length of the shortest admissible line segment $\overline{PQ}$ having P as an endpoint; if no such segment exists, let $d_P = \infty$. Then we define $a(S,T) = \int_{\partial S} \frac{1}{d_P+1} dP$, where the integration is over the border ∂S of S. For example, if S and T are two squares of size $a\times a$ with distance b between them, then $a(S,T)=a(T,S)=\frac{a}{1+b}$. Or, if S is a square of size $a\times a$ located at the center of a square T of size $b\times b$, defining an infinite region $\overline{T}$, then $a(S,\overline{T})=\frac{4a}{1+(b-a)/2}$, so that $a(S,\overline{T})\to 4b$ for $a\to b$.

According to this definition, a border point P of S contributes maximally to a(S,T) if it also lies on the border of T (and the conditions for the case P=Q are met), since in this case $d_P=0$ and $\frac{1}{d_P+1}=1$; and it does not contribute at all if no admissible segment $\overline{PQ}$ exists (e.g., if the border of T is not visible from P), since in this case $d_P=\infty$ and $\frac{1}{d_P+1}=0$. Since $d_P\geq 0$, in any case we have $\frac{1}{d_P+1}\leq 1$; thus $a(S,T)\leq\int_{\partial S} 1\, dP=$ p(S), the perimeter of S. If desired, one can normalize a(S,T) by dividing it by p(S); it then lies between 0 (not at all adjacent) and 1 (maximally adjacent). For example, a hole in a region is maximally adjacent to that region; see Proposition 2.2. We could have used a different function $f(d_P)$, in place of $\frac{1}{d_P+1}$, in defining a(S,T); the essential requirements are that f be a monotonically decreasing function of d_P, and that $f(0)=1$, $f(\infty)=0$.

Proposition 2.1. For regions S and T, a(S,T)=0 if and only if $S\subseteq T$.

Proof: If $S\leq T$ holds, there are no admissible segments $\overline{PQ}$. Note that where the borders touch, the signed normals of S and T point in the same direction. Conversely, if $S \not\subseteq T$, then by the definition of a region, there must exist a border arc of S at every point of which there is an admissible segment, so that $a(S,T) \neq 0$. ||

Proposition 2.2. For regions S and T, $a(S,T)=p(S)$ if and only if either S is bounded, lies inside a hole in T, and its border is identical to the border of that hole; or S is unbounded, T lies inside a hole in S, and the border of S is identical to the outer border of T.

Proof: If these conditions hold, we have $d_P=0$ at every border point of S. Conversely, under no other circumstances can the entire border of S coincide with a part of the border of T. Note that a region cannot consist of several isolated parts; thus it must be contained in one hole only. ||

The definition of $a(S,T)$ can easily be extended to various types of sets other than regions. For example, for a single point P we can define $a(P,T)$ as 0 if $P \in \hat{T}$, and $\frac{1}{d(P,T)+1}$ if $P \notin \hat{T}$, where $d(P,T)$ is the distance from P to T.* (It follows from this definition that $a(P,T)=1$ when P is on the border of T.) Conversely, for a single point Q we can define $a(S,Q)$ using our original definition for sets S and $\{Q\}$; note that here too there are no admissible segments if $Q \in \hat{S}$, but that otherwise $a(S,Q)$ is obtained by integration over the part of the border of S visible from Q. Similarly, we can define

*This definition is not exactly analogous to the one for regions; a single point has zero border length, so that integrating over it should always give zero. The analogy would be better if, in the region definition, we normalized $a(S,T)$ by dividing by $p(S)$.

$a(S,T)$ if S or T is an arc; the details are left to the reader. If $a(S_i,T_j)$ are all defined for $S_1,\ldots,S_m$ and $T_1,\ldots,T_n$, we can also define $a(\bigcup_{i=1}^{m} S_i, \bigcup_{j=1}^{n} T_j)$; again, the details are omitted.

Proposition 2.3. If $T'\subseteq T$ and $S\cap T=\emptyset$, then $a(S,T')\leq a(S,T)$.

Proof: Let $\overline{PQ}$ be any admissible segment in the definition of $a(S,T')$. Let R be the first point in which $\overline{PQ}$ meets T; then $\overline{PR}$ or a shorter line segment is admissible in the definition of $a(S,T)$. (Note that $R\neq P$ since $S\cap T=\emptyset$.) If d_P, d'_P are the lengths of the shortest admissible segments in the definitions of $a(S,T)$ and $a(S,T')$, respectively, we thus have $d_P\leq d'_P$. (Note that for some P's there may be admissible segments with respect to T but not with respect to T'.) Thus $\int_{\partial S}\frac{1}{d'_P+1}\,dP \leq \int_{\partial S}\frac{1}{d_P+1}\,dP$. ||

Proposition 2.4. If $P,P'\notin T$ and $d(P',T)\leq d(P,T)$, then $a(P',T)\geq a(P,T)$, but it is not necessarily true that $a(T,P')\geq a(T,P)$.

Proof: Evidently $a(P',T) = \frac{1}{d(P',T)+1} \geq \frac{1}{d(P,T)+1} = a(P,T)$. On the other hand, let P, P', and T be as shown in Figure 4; then evidently (for P sufficiently close to P') the contributions of side b to $a(T,P)$ and $a(T,P')$ are approximately equal, but sides a and c do not contribute to $a(T,P')$, so that its total contribution is smaller (see example 2.2).

Proposition 2.5. For $P\notin T$, $a(P,T)$ is a continuous function of the position of P, but $a(T,P)$ is not.

Proof: See the proof of Proposition 2.4 and the example given there.

2.2. Fuzzy subsets

Our definition of a(S,T) can be generalized to the case where S and T (call them μ and ν) are bounded fuzzy subsets - that is, functions defined on the plane, with values in [0,1], and equal to zero outside a bounded region $\overline{B}$. The desirability of defining geometric concepts in the fuzzy case, so that they can be measured without having to first crisply segment a picture, is discussed in [2].

We assume in what follows that μ (and ν) are "piecewise constant" in the following sense: We partition $\overline{B}$ into a finite number of regions whose interiors are disjoint, and where the border of each region is contained in the union of the borders of the other regions. Let β be the union of all borders of these regions. In the interior of each region, μ has constant value; at each point of a border, it has one of the neighboring interior values. [Another case of interest is that in which μ and ν are "smooth," i.e., everywhere differentiable; note that we can approximate a piecewise constant μ by a smooth ν which is constant except near the borders, where it changes rapidly from one constant value to another. "Smooth" versions of the definitions in this section could be given, using derivatives in place of differences.]

If $P \neq Q$, we call the segment $\overline{PQ}$ admissible ("with respect to (μ,ν)" understood) if:

a) P is on a border of β, and Q on a border of β. We assume that only two of the constant regions of $\mu(\nu)$ meet at P(Q). (More than two regions met only at a finite number of points, and these can be ignored in defining degree of adjacency.)

b) Let $R \neq P$ be a point of $\overline{PQ}$ such that μ changes value at R as we move from P to Q (there can only be finitely many such R's), and let Δ_R be this change in value. Let the values of μ at the two regions that meet at P be a and b, where b is the value on $\overline{PQ}$ near P, and let $\Delta_P = a-b$. We assume that Δ_P and all the Δ_R's have the same sign, and that $|\Delta_P| > |\Delta_R|$ for all R.

c) Let ∇_R and ∇_Q be defined analogously to Δ_R and Δ_P in (b), with ν replacing μ and the roles of P and Q reversed. We assume that ∇_Q and all the ∇_R's have the same sign; that this is the same sign as in (b); and that $|\nabla_Q| > |\nabla_R|$ for all R.

Conditions (b-c) mean that the changes in μ as we move from P to Q are all in the same direction, and the change "at P" is the largest of them; and analogously for the changes in ν as we move from Q to P. Intuitively, this means that the border points P,Q of β are facing toward or away from each other (since the changes have the same signs in both cases), and that no "stronger" border points (at which larger changes occur) lie between them, so that they are within "line of sight."

It is easily seen that if μ and ν are crisp, these conditions reduce to the definition of admissibility given in Section 2.1 for $P \neq Q$.

When $P=Q$, we call $\overline{PQ}$ admissible if P is a border point of $\mathcal{B}$, and the changes in μ and ν at P (in a fixed direction from one region of the partition of $\overline{B}$ to the other touching it at P) have opposite sign. In this case, let Δ_P and ∇_Q be the changes in the values of μ and ν at P, defined as in the preceding paragraph, where $\Delta_P \cdot \nabla_Q \geq 0$ is assumed.

For each P, let $g(P) = \sup_{\overline{PQ}\text{ admissible}} \frac{\Delta_P \cdot \nabla_Q}{d(P,Q)+1}$, where $d(P,Q)$ is the distance from P to Q. Note that the numerator is always postiive, since the changes in μ and ν at P and Q have the same sign; and that $|\Delta_P|$ and $|\nabla_Q|$ each ≤ 1, so that the numerator is in the interval $(0,1]$. Evidently, in the crisp case the numerator must be 1, and the sup is achieved when the denominator is as small as possible, so that $g(P)$ is the same as $\frac{1}{d_P+1}$ of Section 2.1. It is understood that $g(P)=0$ if no admissible $\overline{PQ}$ exists.

Note that our definition of $g(P)$ involves a tradeoff between the border strengths (=sizes of changes) at P and Q and the distance $d(P,Q)$; the sup may arise from weak changes that are close together (or even coincide), or from stronger changes that are farther apart. The nature of the tradeoff can be manipulated by using some other monotonic function of $d(P,Q)$ in place of $\frac{1}{d(P,Q)+1}$; compare the remark about $f(P)$ in Section 2.1.

Finally, we define $a(\mu,\nu)=\int_\beta g(P)dP$, where the integration is along all borders β of the partition of $\bar{B}$. We leave it to the interested reader to consider extensions of this definition (e.g., to define $a(P,\nu)$ and $a(\mu,Q)$, where P and Q are points), and to investigate the possibility of fuzzy generalizations of the Propositions in Section 2.1. We prove here only

Proposition 2.6. $a(\mu,\nu)\le p(\mu)$, the (fuzzy) perimeter of μ.

Proof: We recall [3] that $p(\mu)$ is just the sum of the lengths of the border arcs of β at which pairs of regions of μ meet, each multiplied by the absolute difference in value between that pair of regions. Now this difference, at a given border point P, is just Δ_P. Thus

$$p(\mu) = \int_\beta |\Delta_P|dP \ge \int_\beta g(P)dP$$

for any ν, since in any case $g(P) = \frac{\Delta_P \cdot \nabla_Q}{d(P,Q)+1}$ for a certain point Q with $|\nabla_Q|\le 1$ and $\frac{1}{d(P,Q)+1} \le 1$, while $\Delta_P \cdot \nabla_Q = |\Delta_P| \cdot |\nabla_Q|$ since they have the same sign. ||

The fuzzy generalization of Proposition 2.3 is false; $\mu \wedge \nu = 0$ and $\nu' \le \nu$ does not imply $a(\mu,\nu') \le a(\mu,\nu)$. In Figure 5, $\overline{PQ}$ is admissible for ν', and $g(P)=\frac{1}{d+n-1}$. With respect to ν, however, the steps in value are all $1/n$, except for the last which is $\frac{2}{n}$; the only possible maximal values of $g(P)$ are thus $\frac{1/n}{d+1}$ and $\frac{2/n}{d+n-1}$. If $n>2$ and $d+n-1<(d+1)n$, these values are both smaller than that for ν'. This counterexample would break down if we used a different definition of $g(P)$, such as $\left[\frac{\Delta_P}{d(P,Q)+1} + \sum_{R\in\overline{PQ}} \left(\frac{\Delta_R}{d(R,Q)+1}\right)\right]\left[\frac{\nabla_Q}{d(Q,P)+1} + \sum_{R\in\overline{QP}} \left(\frac{\nabla_R}{d(R,P)+1}\right)\right]$; but we will not pursue this alternative approach here.

2.3. Digital polygons

Subsets of digital pictures may be considered from different points of view, e.g., as sets of grid points, as sets of cells, or as digital polygons. For purposes of defining quantitative adjacency in the digital (crisp) case, it is convenient to deal with digital polygons.

In a digital simple polygon $S = \langle P_0, P_1, \ldots, P_n \rangle$, for $k = 0,1,\ldots,n$, the P_k are all grid points with integer coordinates; points P_k and P_{k+1} are 8-neighbors (for $P_{n+1}=P_0$); in relation to the interior of S the sequence $P_0, P_1, \ldots, P_n$ has clockwise orientation; and the border of S is non-crossing, i.e., S is a simple polygon in the usual sense. Because of the fixed orientation, finite and infinite digital simple polygons can both be defined in this way. For $S = \langle P_0, P_1, \ldots, P_n \rangle$, the complementary polygon $\bar{S}$ is given by $\langle P_n, P_{n-1}, \ldots, P_0 \rangle$.

Because digital simple polygons are regions as defined in Section 2.1, the degree of adjacency a(S,T) is defined for digital polygons S and T. However, for the needs of picture processing or computer graphics, a more specifically digital approach will be used. For this purpose, the set of border points BP(S) of a digital polygon S will be restricted to grid points on the (real) border of S, i.e., $BP(S)=\{P_0, P_1, \ldots, P_n\}$ for $S = \langle P_0, P_1, \ldots, P_n \rangle$. Now, admissible line segments are defined as in Section 2.1 for $P \in BP(S)$ and $Q \in BP(T)$, for the digital polygons S and T. (At a vertex of a digital polygon, the (signed) normal is defined to be the bisector of the vertex angle.). Let AL(S,T) be the set of all admissible line segments, from border points of S to border points of T. Note that $AL(S,T) \neq AL(T,S)$ for almost all digital polygons S and T; $AL(S,\bar{S})=BP(S)$, and $AL(S,S)=\emptyset$ (the empty set). For digital polygons S and T, the digital degree of adjacency is defined to be

$$a_{dig}(S,T) = \begin{cases} \sum_{P \in BP(S)} 1/(1+d_P), & \text{if } AL(S,T) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

Note that all the properties given in Section 2.1 for function a are true for a_{dig} too. The property $a(S,T) \leq p(S)$, the perimeter of S, is replaced by $a_{dig}(S,T) \leq \text{card } BP(S)$, which may be considered to be the digital perimeter of S.

Proposition 2.7. For digital polygons S,T, $a_{dig}(S,T) = \text{card } BP(S)$ if and only if $T=\bar{S}$.

Proof: Because of $AL(S,\bar{S})=BP(S)$ we have $d_P=0$ for all $P \in BP(S)$. Thus, $a_{dig}(S,\bar{S}) = \text{card } BP(S)$. On the other hand, if $T \neq \bar{S}$ then there exists at least one point $P \in BP(S)$ with $d_P > 0$.

The normalized degree of adjacency is defined by

$$a^*_{dig}(S,T) = a_{dig}(S,T)/\text{card } BP(S),$$

for digital polygons S and T. The behavior of this concept of degree of adjacency is illustrated by the following two examples.

Example 2.1. Assume two convex digital polygons S and T_n with distance n between them, as shown in Figure 6. For different values of n, polygon T_n changes its position in relation to S. For example, for n=0, S is in centralized position within T_0, and for n=7 and n= -7, S and T_n are in touching positions. By symmetry, for n=0,1,2,... we have $a_{dig}(S,T_n)=a_{dig}(S,T_{-n})$ and $a_{dig}(T_n,S)=a_{dig}(T_{-n},S)$. For the normalized degrees of adjacency we use card BP(S)=16 and card BP(T_n)=24. For n=0,1,2,...,10, the values of $a^*_{dig}(S,T_n)$ and $a^*_{dig}(T_n,S)$ are given in Table 1. These values are graphically illustrated in Figure 7. As seen in this Figure, there is a somewhat unbalanced behavior of the proposed measure a^*_{dig} for intersecting positions ($-6 \le n \le 6$) and non-intersecting positions ($|n| \ge 8$) of the two polygons. Even for the "most adjacent" positions (n=7) we don't have the maximum value, which is realized for $|n|=2$. This behavior is due to the influence of border points for which P=Q, $P \in BP(S)$ and $Q \in BP(T_n)$. We may change our definition of a^*_{dig} by requiring $d_P=0$ if and only if there are border segments PP' and QQ' of S and T_n, respectively, such that $P \neq P'$ (non-trivial segment), PP'=QQ', and the signed normals of S and T_n on this common border segment point in exactly inverse directions. In all other cases of P=Q, $P \in BP(S)$ and $Q \in BP(T_n)$, we let $d_P=\infty$. The resulting modified function a^*_{dig} is denoted by a^+_{dig}. Results for a^+_{dig} are also shown in Table 1 for $-10 \le n \le 10$.

Remark. Convex digital polygons are restricted to be octagons at most, where "convexity" is understood as in the real plane. Then, for two convex digital polygons S and T with N=card BP(S) + card BP(T), there exists an O(N) worst case time algorithm for computing $a_{dig}(S,T)$ or $a^{*}_{dig}(S,T)$. The basic ideas of the algorithm are as follows:

(i) Determine the upper and lower tangents on $S \cup T$ dividing the polygonal border of S (or T) into a connected part, where no admissible line segments to T (or to S) are possible, and a second connected part containing all points which may contribute to $a_{dig}(S,T)$ (or to $a_{dig}(T,S)$).

(ii) Perform two search procedures, one top-down and one bottom-up, to compute candidate values for d_P, for all points P in the interesting part of the border of S, where only points in the interesting part of the border of T have to be considered. In both search procedures the connection line between two points $P \in BP(S)$ and $Q \in BP(T)$ which are under consideration for computing d_P moves monotonically down (or up) in the interesting parts of the borders of S and T. During these search procedures at most two crossings of the borders of S and T are possible.

(iii) For all points P in the interesting part of the border of S take the minimum of both candidate values found in (ii) to compute d_P.

(iv) Determine $a_{dig}(S,T)$ by using the values of d_P.

Using small examples, it can already ben seen that in fact two search procedures are necessary in step (ii) which cooperate to give the final result in step (iv).

Example 2.2. In this example, we consider a moving point in relation to a fixed digital polygon T; see Proposition 2.4. P_n denotes the point at distance n from T as illustrated in Figure 8. In this case, it follows that $a_{dig}(P_n,T)=1/(n+1)$ for $n\geq 0$, and $a_{dig}(T,P_0)=1$, $a_{dig}(T,P_1)=1.94646$, $a_{dig}(T,P_2)=1.47377$, $a_{dig}(T,P_3)=1.99526$, etc.

3. Quantitative surroundedness

In Sections 3.1 and 3.2 we present two quantitative definitions of surroundedness in the Euclidean plane, and also indicate how each of them generalizes to the fuzzy case. Section 3.3 discusses quantitative surroundedness in digital pictures.

3.1. Visual surroundedness

Let P be a point and T a (bounded) set. Let $r_\theta(P,T)=1$ if the ray emanating from P in direction θ meets T, and $r_\theta(P,T)=0$ otherwise. We define the degree of visual surroundedness of P by T as $v(P,T)=\frac{1}{2\pi}\int_0^{2\pi} r_\theta(P,T)d\theta$. (This integral might not be defined if T is some type of pathological set; but it is evidently defined for various types of well-behaved sets, such as regions and arcs.)

If S is a (well-behaved) set, we define v(S,T) as $\min_{P\in S} v(P,T)$. (Another possibility would be to take the "average" value of v(P,T) for all $P\in S$.) It follows that v(S,T) is defined by a border point of S, i.e., $v(S,T) = \min_{P\in\partial S} v(P,T)$. For the sets given in Figure 9, we have $v(S,T)=v(P,T)=\tan^{-1}(5/6)/\pi=0.2211$ and $v(T,S)=v(Q,S)=\tan^{-1}(1/2)/\pi=0.1476$.

If S and T are fuzzy sets, call them μ and ν, we define $r_\theta(P,\mu,\nu)=1$ if $\nu(R)\geq\mu(P)$ at some point R on the ray emanating from P in direction θ. (Recall [2] that ν surrounds μ if, for any point P and any path π from P to B, there exists $R\in\pi$

such that $\nu(R) \geq \mu(P)$.) We then define $v(P,\mu,\nu)$ as $\frac{1}{2\pi} \int_0^{2\pi} r_\theta(P,\mu,\nu)\,d\theta$, and we define $v(\mu,\nu)$ by taking the min over all P in the plane. (In the case of taking the "average" value of $v(P,\mu,\nu)$ the denominator for the average is $\int_B \mu\,dP$.) Evidently this generalizes the crisp definition.

<u>Proposition 3.1</u>. If $T \supseteq T'$, then $v(P,T) \geq v(P,T')$ for any P and $v(S,T) \geq v(S,T')$ for any S.

<u>Proof</u>: This follows immediately from the fact that $r_\theta(P,T) \geq r_\theta(P,T')$ for any P. Analogously, in the fuzzy case, if $\nu \geq \nu'$, then $v(P,\mu,\nu) \geq v(P,\mu,\nu')$ for any P and μ. ||

It is not hard to see that $v(P,T)$ is a continuous function of the position of P. On the other hand, $v(P,T)$ need not increase as P moves closer to T, even if T is convex, as illustrated in Figure 10.

If T subtends angle α from P, we evidently have $v(P,T) = \alpha/2\pi$; thus as P approaches T, if T is convex $v(P,T)$ approaches 1/2 since α approaches π. Note that when $P \in T$ we have $v(P,T)=1$.

3.2. Topological surroundedness

Even if $v(P,T)=1$, T may not surround P in the usual sense, since there may be a curved path from P to B (the "background" region, outside the picture) that does not intersect T, as illustrated in Figure 11. In this section we introduce an alternative definition of quantitative surroundedness that is more closely related to the usual (topological) definition.

Intuitively, the degree to which T (topologically) surrounds P is related to how much a path from P must change direction in order to reach B without intersecting T. For example, if T is a spiral, and P is "surrounded" by a very large number of turns of T, a path from P that does not intersect T must turn through a very large multiple of 2π before it can reach B.

Let π_θ be any rectifiable path from P' to B, starting at P' in direction θ, that does not intersect T. (If no such path exists, we define $t(P',T)=\infty$.) Let $C_{\pi_\theta}(P',T)=\int_{\pi_\theta} |c_{\pi_\theta}(P)|dP$, where $c_{\pi_\theta}(P)$ denotes the absolute curvature of π_θ at an arbitrary point P on π_θ. Let $C_\theta(P',T)=\inf_{\pi_\theta} C_{\pi_\theta}(P',T)$; if T is a "well behaved" set (e.g., a region) and $P'\notin T$ and P' is not inside a hole of T, $C_\theta(P',T)$ will be finite. Finally, let $t(P',T)=\frac{1}{2\pi}\int_0^{2\pi} C_\theta(P',T)d\theta$, the average of $C_\theta(P',T)$ over θ. [Alternatively, we could have used $\inf_\theta C_\theta(P',T)$, but using the average allows our definition to be sensitive to "partial" surroundedness of P' by T. For example, if T is a circle with a small gap, and P' is at its center, there exists a direction in which π_θ does not have to turn at all, so that the inf definition gives 0, as if T were not there at all. On the other hand, the averaging definition reflects the fact that some paths may have to turn by as much as π before they can get out of T; in fact, the average is approximately $\frac{\pi}{2}$, but it gets smaller as the gap in the circle gets wider.]

If S is a ("well behaved") set, we define t(S,T) as the inf of t(P,T) for all $P\in\partial S$. If S and T are fuzzy (call them μ and ν), we use exactly analogous definitions, except that π_θ is a path from P to B such that $\nu(R)<\mu(P)$ for all R on π_θ; this is the fuzzy version of "does not intersect T". In the fuzzy case, $t(\mu,\nu)$ would be $\frac{\iint t(P,\mu,\nu)dxdy}{\iint \mu dxdy}$ if we use the averaging definition.

Proposition 3.2. If $T\supseteq T'$, then $t(P,T)\geq t(P,T')$ for any P, and $t(S,T)\geq t(S,T')$ for any S.

Proof: This follows immediately from the fact that any path (from any P) that meets T' also meets T. Analogously, in the fuzzy case, if $\nu\geq\nu'$, then $t(\mu,\nu)\geq t(\mu,\nu')$ for any μ. //

It is not hard to see that t(P,T) is a continuous function of the position of P. However, t(P,T) need not increase as P moves closer to T, even if T is convex, as we see from the examples in Figure 10.

Let T be convex and subtend angle α at P. It is easily seen that for all θ outside that angular sector, paths from P to B exist that do not turn at all and do not meet T; but if θ is inside the sector, say δ away from the nearer boundary of the sector, a path from P in direction θ must turn by at least δ in order to reach B without meeting T. Moreover, such paths exist that do not turn by more than δ. It follows that t(P,T) is just the average value of β for all directions θ in the sector; this is evidently just $\alpha/2$. In particular, as P approaches T, $t(P,T)$ approaches $\pi/2$, since α approaches π. Note that when $P \in T$ we have $t(P,T)=\infty$.

3.3. Surroundedness for digital polygons

For subsets of digital pictures our approaches to quantitative surroundedness have to be "digitized."

In the case of visual surroundedness we define $v_{dig}(S,T) = \min_{P \in BP(S)} v_{dig}(P,T)$ for digital polygons S and T, where $v_{dig}(P,T) = v(P,T) = \alpha/2\pi$ if T subtends angle α from P. The rectangles S and T in Figure 9 may be considered to be digital polygons, for example. Then, the values of $v_{dig}(S,T)$ and $v_{dig}(T,S)$ remain the same as given in Section 3.1 by v(S,T) and v(T,S), respectively. The straightforward approach to computing $v_{dig}(S,T)$ would be:

```
angle = +∞,
compute the convex hulls S',T' of S,T using any
 desired linear time algorithm
for all points P in BP(S') do
   1. compute the two tangents from P to T' and
      the angle α between these tangents,
   2. if angle >α then angle = α od
return  angle/2π.
```

Since when P moves around S' the related tangential points $Q_1, Q_2 \in BP(T')$ move around T' monotonically, this straightforward algorithm leads to an O(N) time algorithm, N=card BP(S) + card BP(T), by using two points to the actual tangential points in BP(T').

In the case of topolcgical surroundedness, besides the restriction of ∂S to BP(S) we have to digitize the set of possible directions [illegible] for paths from BP(S) to the background B, for a digital polygon S. Let us assume that [illegible] is restricted to the set $ang_m = \{n \cdot \frac{2\pi}{m} : n=0,1,2,\ldots,m-1\}$, for $m \geq 1$. Then $C_\theta(P,T)$ denotes the minimal angle that a path π_θ in direction θ starting at P may take around T to B, as defined in Section 3.2, and $t^m_{dig}(P,T)$ is defined by $\frac{1}{m} \sum_{\theta \in ang_m} C_\theta(P,T)$, for a digital polygon T and a point P. Finally, we have $t^m_{dig}(S,T) = \min_{P \in BP(S)} t^m_{dig}(P,T)$. Obviously, the computational requirements for computing $t^m_{dig}(S,T)$ exceed those for computing the visual surroundedness measure $v_{dig}(S,T)$, but still t^m_{dig} seems to be a practically useful function. For example, in the situation of Figure 9 we have $t^8_{dig}(P,T) = \frac{1}{8}(\tan^{-1}(5/6)+0+0+0+0+0+0+0) = 0.0276 \cdot \pi = 0.0868$. It follows that $t^8_{dig}(S,T) \leq t^8_{dig}(P,T) = 0.0868$. Analogously, $t^8_{dig}(T,S) \leq t^8_{dig}(Q,S) = 0.0184 \cdot \pi = 0.0579$. For computing $t^m_{dig}(S,T)$ nearly the same algorithm may be used as for computing $v_{dig}(S,T)$, but with some extensions. After computing the two tangents from P to T' we determine $\alpha = t^m_{dig}(P,T')$ by using the minimal angular differences to these tangents if θ is between these tangents; otherwise $C_\theta(P,T')=0$. Thus, $t^m_{dig}(S,T)$ with $N = \text{card } BP(S) + \text{card } BP(T)$ may be computed within $O(mN)$ time in the worst case sense.

4. Concluding remarks

We have given definitions for quantitative adjacency and surroundedness for crisp and fuzzy sets in the real plane, and also for crisp digital polygons; we have not considered the digital fuzzy case. We have briefly described some algorithms for computing these quantities in the digital case, but many issues have been left open; for example, fast algorithms for the adjacency measure in the general case (arbitrary polygons) are still an open problem.

The proposed measures should be of interest in the study of stochastic geometry in the real plane. At the same time, these measures can be used to characterize relationships between objects in a segmented digital picture, or to compare objects in two different pictures.

References

1. A. Rosenfeld and A. C. Kak, Digital Picture Processing, Academic Press, New York, 1982, Chapter 11.

2. A. Rosenfeld, The fuzzy geometry of image subsets, CAR-TR-14, CS-TR-1299, Center for Automation Research, University of Maryland, College Park, MD.

3. A. Rosenfeld and S. Haber, The perimeter of a fuzzy set, CAR-TR-8, CS-TR-1286, Center for Automation Research, University of Maryland, College Park, MD.

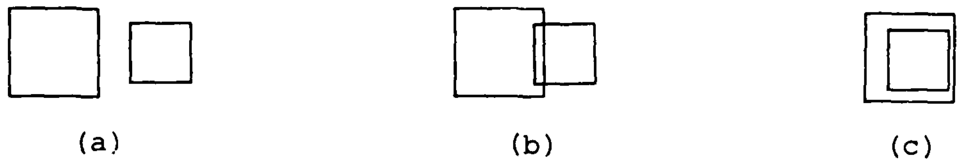

Figure 1. Examples of near-adjacency (a,b) and non-adjacency (c).

Figure 2. The line-of-sight requirement in measuring adjacency.

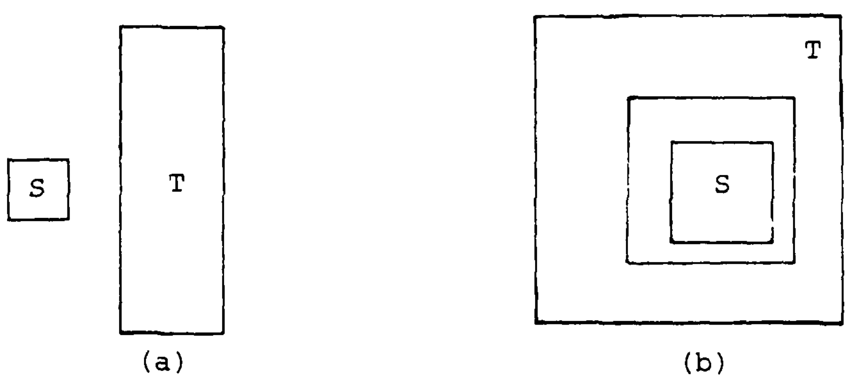

Figure 3. Degree of adjacency is not symmetric.

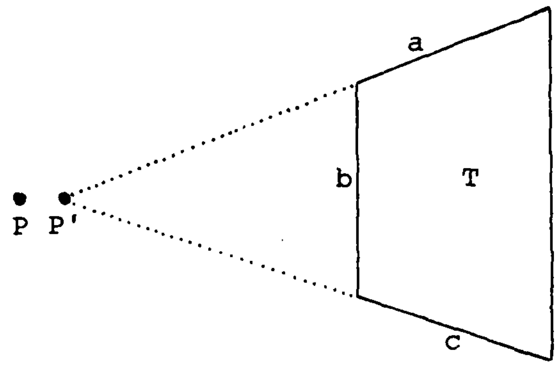

Figure 4. The degree of adjacency of a region T to a point P is not necessarily a monotonically decreasing function of d(P,T), and is not necessarily a continuous function of the position of P.

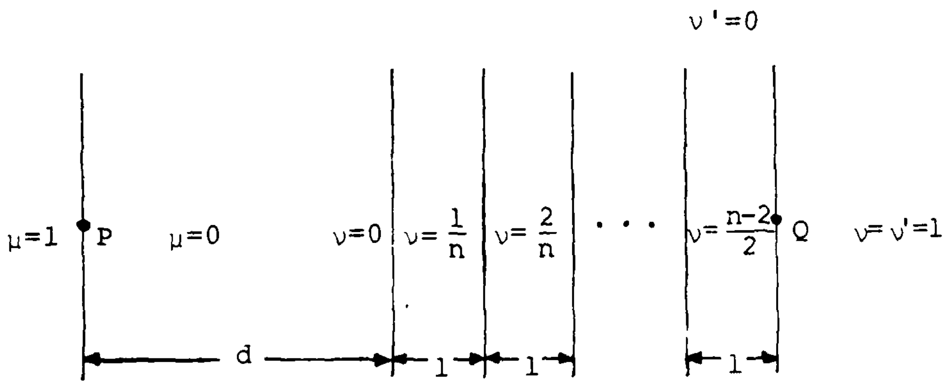

Figure 5. Counterexample to the fuzzy generalization of Proposition 3.

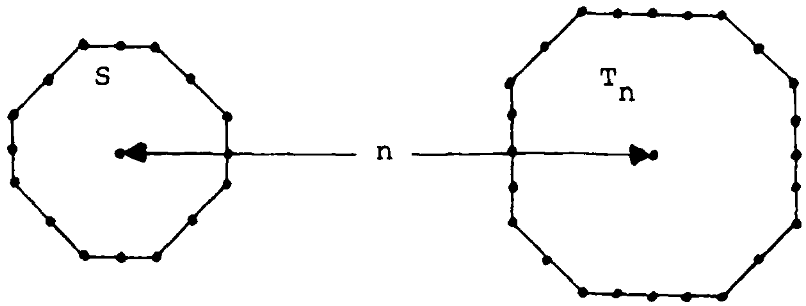

Figure 6. Two convex digital polygons n units apart (measured between the two center points).

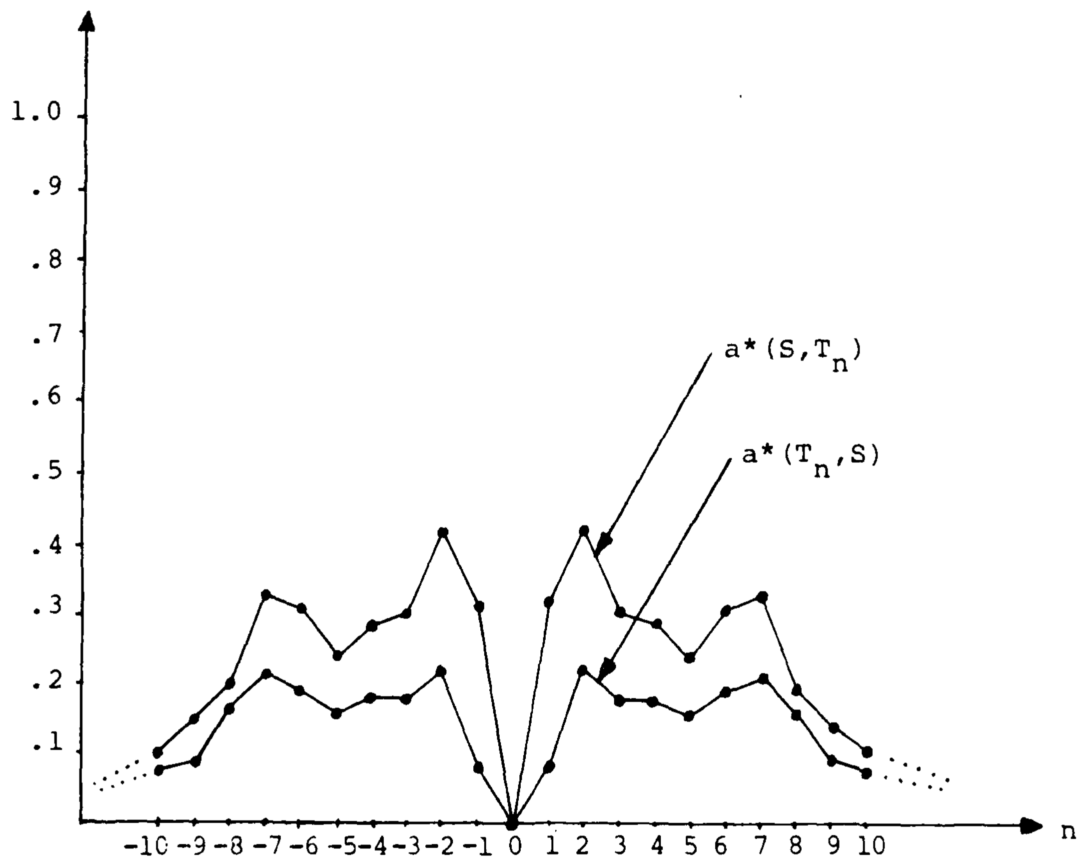

Figure 7. Functions $a^*_{dig}(S,T_n)$ and $a^*_{dig}(T_n,S)$ for the polygon in Figure 6, for $-10 \le n \le +10$.

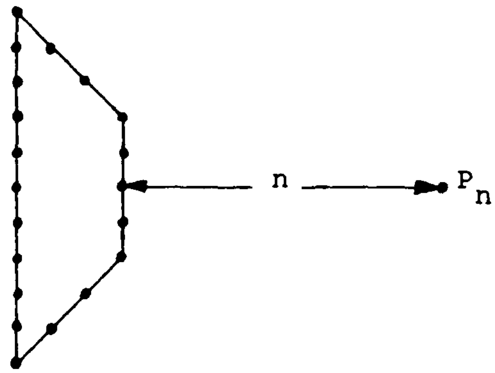

Figure 8. Point P_n at distance n from polygon T as used in Example 2.2.

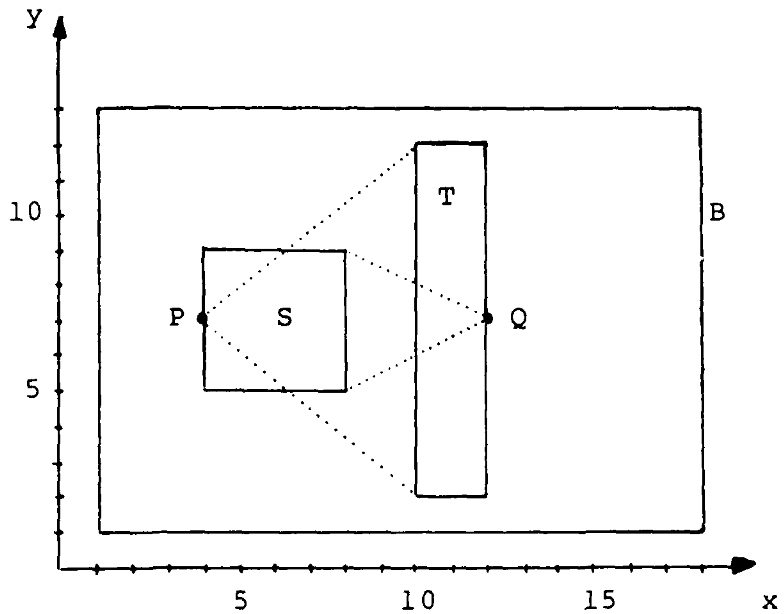

Figure 9. Example sets for illustrating surroundedness.

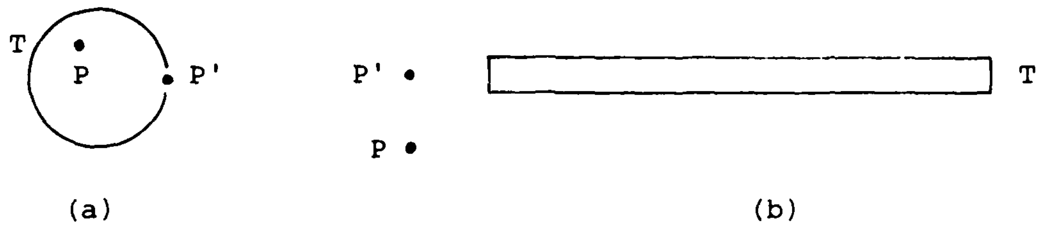

Figure 10. P' is closer to T than P, but v(P,T)>v(P',T).

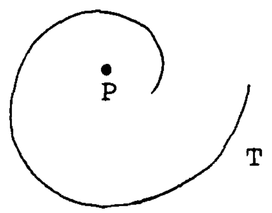

Figure 11. Visual surroundedness does not imply surroundedness.

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